$$T(n) = T(2n/5) + T(4n/5) + n$$

$$\underbrace{n + \left(\frac{6}{5}\right) n + \left(\frac{6}{5}\right)^2 n + \cdots +}$$

n — n

$\frac{2n}{5}$ $\frac{4n}{5}$ — $\frac{6}{5} n$

$\left(\frac{2}{5}\right)^2 n$ $\left(\frac{2}{5}\right)\left(\frac{4}{5}\right) n$ $\left(\frac{2}{5}\right)\left(\frac{4}{5}\right) n$ $\left(\frac{4}{5}\right)^2 n$ — $\left(\frac{6}{5}\right)^2 n$

$\cdots$ — $\left(\frac{6}{5}\right)^3 n$

$$\frac{4}{25} + \frac{8}{25} + \frac{8}{25} + \frac{16}{25} = \frac{36}{25}$$

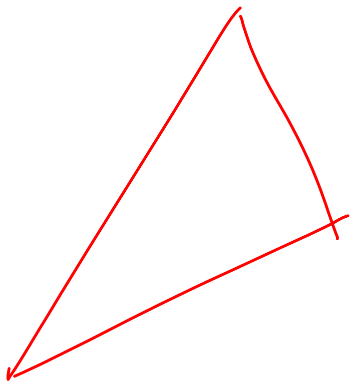
$\log_{\frac{5}{2}} n$

$\log_{\frac{5}{4}} n$

$$T(n) \le n + \left(\frac{6}{5}\right)n + \left(\frac{6}{5}\right)^2 n + \dots + \left(\frac{6}{5}\right)^{\log_{\frac{5}{2}} n} \cdot n$$

$$= n \cdot \left(\sum_{i=0}^{\lfloor \log_{\frac{5}{2}} n \rfloor} \left(\frac{6}{5}\right)^{i} \right) = n \, \frac{\left(\frac{6}{5}\right)^{\lfloor \log_{\frac{5}{2}} n \rfloor + 1} - 1}{\left(\frac{6}{5}\right) - 1}$$

$$\le n \cdot \left(\frac{6}{5}\right)^{\log_{\frac{5}{2}} n + 1} = n \cdot \left(\frac{6}{5}\right)^{\log_{\frac{5}{2}} n} \cdot \left(\frac{6}{5}\right)$$



$$= n \cdot n^{\log_{\frac{5}{2}} \frac{6}{5}}$$

$$= n^{1 + \log_{\frac{5}{2}} \frac{6}{5}} = n^{\log_{\frac{5}{2}} \left(\cancel{\frac{5}{2}} \cdot \cancel{\frac{6}{5}}^{\,3} \right)}$$

$$n^{\log_{\frac{5}{4}} \left(\cancel{\frac{5}{4}} \cdot \frac{\cancel{6}^{\,3}}{\cancel{5}\;2} \right)}$$

$$= n^{\log_{\frac{5}{2}} 3}$$

$$T(n) = \Omega\left(n^{\log_{\frac{5}{4}} \frac{3}{2}} \right)$$

$$T(n) = O\left(n^{\log_{\frac{5}{2}} 3} \right)$$

$$T(n) = T(n/2) + \sqrt{n}$$

$\lg_2 1 = 0 < \frac{1}{2} \Rightarrow T(n) = \Theta(\sqrt{n})$

$$T(n) = 3T(n/2) + n \log n$$

$\lg_2 3 > 1 \Rightarrow T(n) = \Theta(n^{\lg_2 3})$

$$T(n) = T(2n/5) + T(4n/5) + n$$

Data l'equazione di ricorrenza $T(n) = 9T(n/a) + n$, per quale valore del parametro a la sua soluzione **non** è $O(n^2)$

$$T(n) = 2T(n-1) + n$$

$$T(n) = 4T(n/2) + n \log n$$

$T(n)= T(2n/5)+T(3n/5) + n$

$T(n)= T(n/3)+n/2$

$T(n)= 2T(n/2) + 3T(n/3) + n$

$T(n)= 8T(n/2)+n^2 \log n$

$T(n)= 3T(n/2) + T(n/3) + n^2$

$T(n)= 8T(n/2)+n^3 \log n$

$$T(n) = 2T(n-1) + n$$

$$= 2(2T(n-2) + n - 1) + n$$

$n \longrightarrow n$

$n-1 \quad n-1 \longrightarrow 2(n-1)$

$T(n-2) \quad T(n-2) \quad T(n-2) \quad T(n-2) \longrightarrow 4(n-2)$

$$n + 2(n-1) + 2^2(n-2) + 2^3(n-3) + \dots + 2^{n-1}(n-(n-1))$$

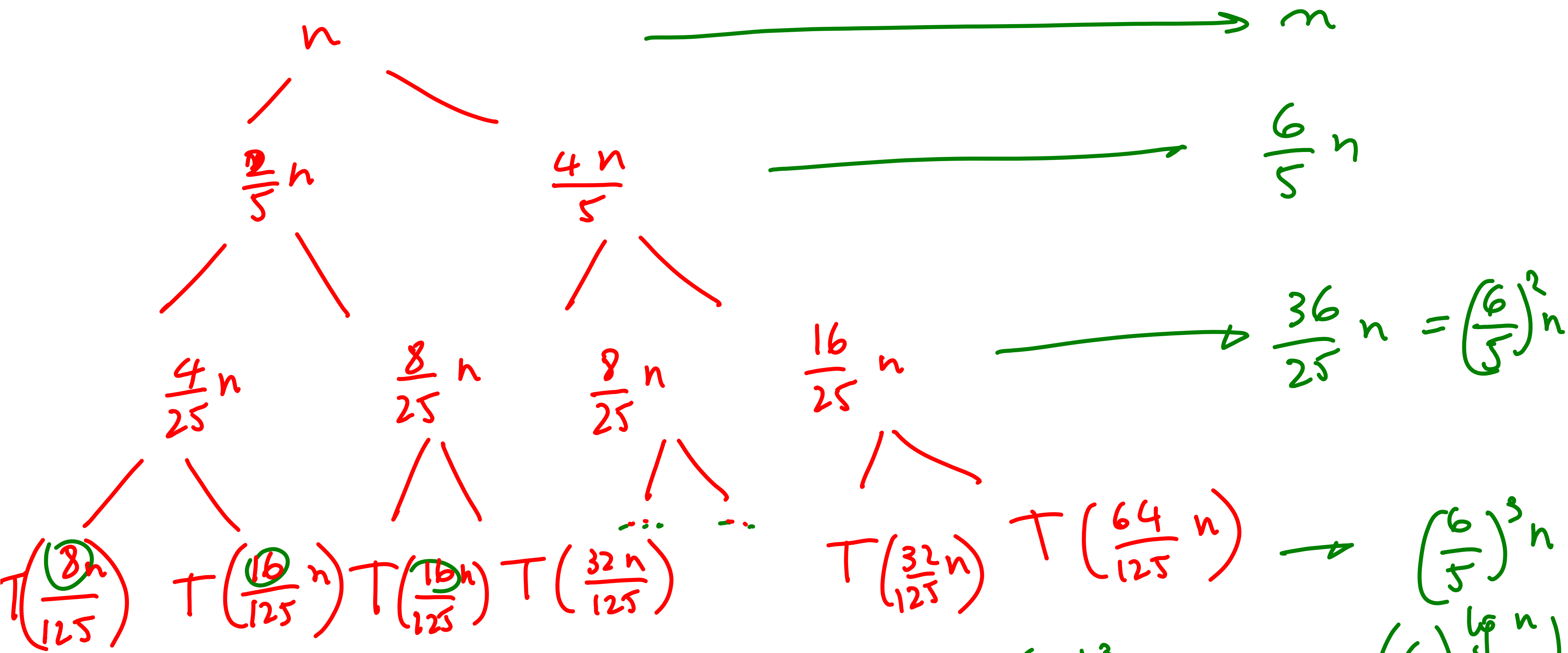
$$n + 2n + 2^2 n + \dots + 2^{n-1} \cdot n$$

$$-\left(2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + (n-1) \cdot 2^{n-1}\right)$$

$$\sum_{i=1}^{n-1} i \cdot 2^i \qquad \sum_{i=1}^{n} 2^i$$

$$T(n) = T(2n/5) + T(4n/5) + n$$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



$$T(n) = \Theta\left(n + \frac{6}{5}n + \left(\frac{6}{5}\right)^2 n + \left(\frac{6}{5}\right)^3 n + \dots + \left(\frac{6}{5}\right)^{\lg n} n\right)$$

$$n \cdot \sum_{i=0}^{\lg n} \left(\frac{6}{5}\right)^i = n \cdot \frac{\left(\frac{6}{5}\right)^{\lg n + 1} - 1}{\frac{6}{5} - 1} = \Theta\left(n \cdot \left(\frac{6}{5}\right)^{\lg n}\right)$$

$$\boxed{\lg_{\frac{5}{2}} n}$$

$$\boxed{\lg_{\frac{5}{4}} n}$$

$$\uparrow$$

$$\left(\frac{6}{5}\right)^{\lg_{\frac{6}{5}} n} \cdot \lg_2 \frac{6}{5}$$

$$= n^{\lg_2 \frac{6}{5}}$$

$$n \cdot \left(\frac{6}{5}\right)^{\lg_k n}$$

$$\frac{5}{2} < k \leq \frac{5}{4}$$

$$\underbrace{T(n) = 6T\left(\frac{n}{6}\right) + n \log n}$$

$a = 6 \,, \; b = 6$ $\qquad n^{\log_6 6}$ $\qquad n \log n$

$$n \log n \neq \Omega\left(n^{1+\varepsilon}\right)$$

Data l'equazione di ricorrenza $T(n)= 9T(n/\cancel{a}\,b) + n$, per ~~quale~~ quali ~~valore~~ valori del parametro ~~a~~ b la sua soluzione **non** è $O(n^2)$

$$T(n) = 9T\left(\frac{n}{b}\right) + n$$

$$n = O\left(n^{\log_b 9 - \varepsilon}\right) \Longrightarrow T(n) = \Theta\left(n^{\log_b 9}\right)$$

$$\Updownarrow$$

$\log_b 9 - \varepsilon \geq 1$, per qualche $\varepsilon > 0$

$$\Updownarrow$$

$$\log_b 9 > 1$$

$$\Updownarrow$$

$$9 > b$$

$$n = \Theta(n^{\log_b 9}) \implies T(n) = \Theta(n^{\log_b 9} \cdot \lg n)$$

$$\Updownarrow$$

$$1 = \log_b 9 \iff b = 9$$

$$n = \Omega(n^{\log_b 9 + \varepsilon}) \iff 1 > \log_b 9 \iff \boxed{b > 9}$$

$$a f\left(\frac{n}{b}\right) \leq c f(n) \qquad 0 < c < 1 \qquad c = \frac{9}{b} < 1$$

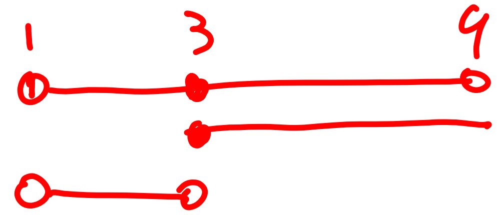
$$9 \frac{n}{b} = \frac{9}{b} n, \quad \text{O.K.}$$

$$\implies T(n) = \Theta(n)$$

$$
T(n) = \begin{cases} \Theta\left(n^{\log_b 9}\right) & \text{se } 1 < b < 9 \\ \Theta(n \log n) & \text{se } b = 9 \\ \Theta(n) & \text{se } b > 9 \end{cases}
$$

$b < 9$, $\quad T(n) = O(n^2) \iff \log_b 9 \leq 2$

$\iff 9 \leq b^2 \iff 9 > b \geq 3$



$\boxed{1 < b < 3} \Leftarrow$ soluzione